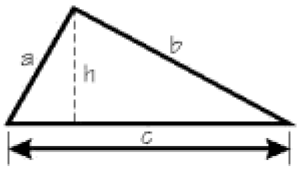
APPENDIX B – GEOMETRIC AND TRIGONOMETRIC REFERENCE CHARTS

The charts in this section contain a number of formulas for performing geometric and trigonometric calculations. These charts should be used as references in calculating quantities for measurement and payment.

OBLIQUE TRIANGLE



h = altitude

c = base

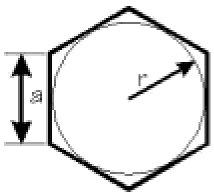
$s = \frac{a + b + c}{2}$

$$A = \frac{ch}{2}$$

or,

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

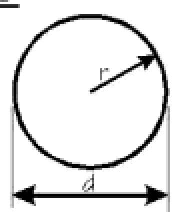
REGULAR POLYGON



Example:
Polygon (hexagon) with circle inscribed within.

$$A = \frac{\text{Sum of sides X r}}{2}$$

CIRCLE



r = radius $r = \frac{\text{circum}}{2\pi}$

d = diameter $d = \frac{\text{circum}}{\pi}$

$\pi = 3.1416$ circum $= \pi d$

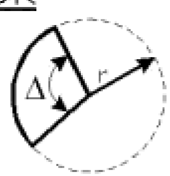
$$A = \pi r^2$$

$$A = .7854\, d^2$$

$$A = \frac{\text{circum X r}}{2}$$

$$A = \frac{\pi d^2}{4}$$

SECTOR

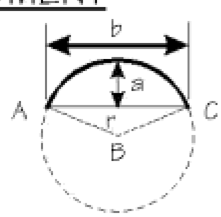


r = radius

Δ = Degrees of Enclosed Angle

$$A = \frac{\text{Area of Circle X } \Delta}{360^\circ}$$

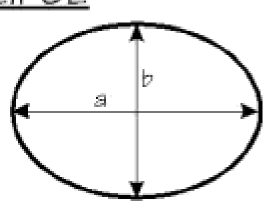
SEGMENT



A = 2/3 ab (Approximately)
Use for segments having M.O. of less than 1/4 the length of the chord

A=0.7854ab
Use for segments

ELLIPSE

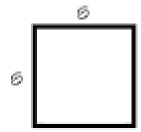
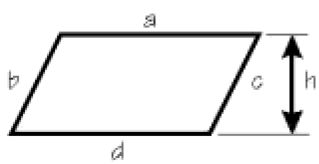
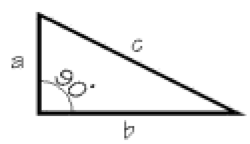
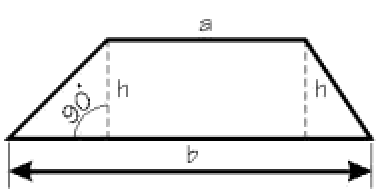
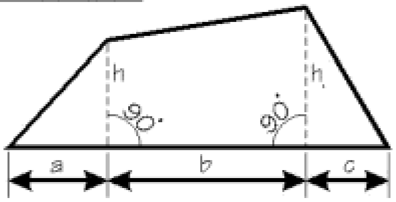


a = Major Axis

b = Minor Axis

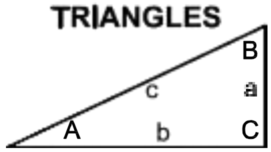
$$A = .7854\, ab$$

Areas of Common Geometric Shapes

SQUARE 	All Angles = 90° $s = s$	$A = s^2$
RECTANGLE 	All Angles = 90°	$A = ah$
PARALLELOGRAM 	$a = d$ $b = c$ Opposite Sides are Parallel	$A = ah$
RIGHT TRIANGLE 	$a^2 + b^2 = c^2$	$A = \frac{ab}{2}$
TRAPEZOID 	a parallel to b	$A = \frac{(a + b)}{2} h$ or Solve as Sum of Rectangle and 2 Triangles
TRAPEZIUM 	h parallel to h_1	$A = \frac{b(h + h_1) + ah + ch_1}{2}$

Areas of Common Geometric Shapes

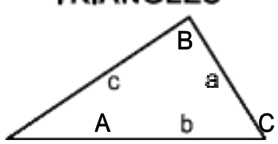
RIGHT ANGLED TRIANGLES



$$a^2 = c^2 - b^2$$
$$b^2 = c^2 - a^2$$
$$c^2 = a^2 + b^2$$

Known	Required					
	A	B	a	b	c	Area
a, b	$\tan A = \frac{a}{b}$	$\tan B = \frac{b}{a}$			$\sqrt{a^2 + b^2}$	$\frac{ab}{2}$
a, c	$\sin A = \frac{a}{c}$	$\cos B = \frac{a}{c}$		$\sqrt{c^2 - a^2}$		$\frac{a\sqrt{a^2 + b^2}}{2}$
A, a		$90^\circ - A$			$\frac{a}{\sin A}$	$\frac{a^2}{2 \tan A}$
A, b		$90^\circ - A$	$b \tan A$		$\frac{b}{\cos A}$	$\frac{b^2 \tan A}{2}$
A, c		$90^\circ - A$	$c \sin A$	$c \cos A$		$\frac{c^2 \sin 2A}{4}$

OBLIQUE ANGLED TRIANGLES



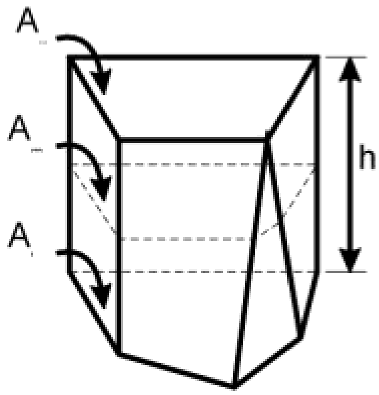
$$a^2 = b^2 + c^2 - 2\,bc \cos A$$
$$b^2 = a^2 + c^2 - 2\,ac \cos B$$
$$c^2 = a^2 + b^2 - 2\,ab \cos C$$

$$s = \frac{a+b+c}{2}$$

Known	Required					
	A	B	C	b	c	Area
a, b, c	$\cos A = \frac{b^2+c^2-a^2}{2bc}$	$\cos B = \frac{a^2+c^2-b^2}{2ac}$	$\cos C = \frac{a^2+b^2-c^2}{2ab}$			$\sqrt{s(s-a)(s-b)(s-c)}$
a, A, B			$180^\circ - (A+B)$	$\frac{a \sin B}{\sin A}$	$\frac{a \sin C}{\sin A}$	
a, b, A		$\sin B = \frac{b \sin A}{a}$			$\frac{b \sin C}{\sin B}$	
a, b, C	$\tan A = \frac{a \sin C}{b - a \cos C}$				$\sqrt{a^2 + b^2 - 2\,ab \cos C}$	$\frac{ab \sin C}{2}$

Trigonometric Values for Right and Oblique Triangles

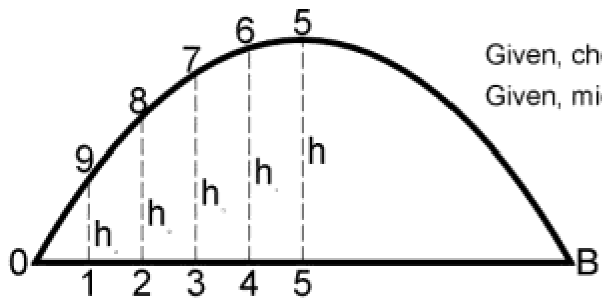
PRISMOIDAL FORMULA:



1. Prismoid has parallel bases.
2. Middle area is <u>not</u> ½ sum of upper and lower base areas.
3. Average end areas give slightly larger answers.
4. To use formula, upper and lower bases will be cross sections.

$$\text{Volume} = (A + A + 4A)\frac{h}{6}$$

PARABOLA



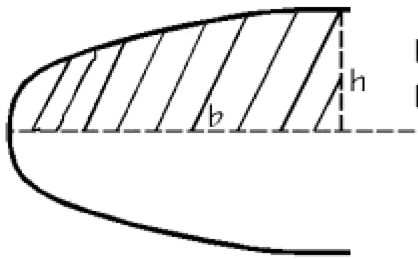
Given, chord = OB

Given, middle ordinate = h

To draw a parabola to scale, number the equally spaced ordinates between the end and center as shown; then, $h = (h)\frac{9x1}{5x5}$, $h = (h)\frac{8x2}{5x5}$, etc.

Prismoidal Volume and Parabolic Height Calculations

PARABOLA



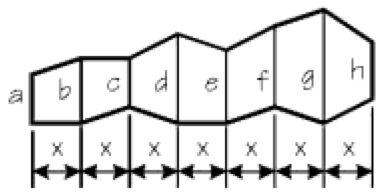
h = height
b = base

Area of Shaded Portion:

$$A = \frac{2bh}{3}$$

(Approximately)

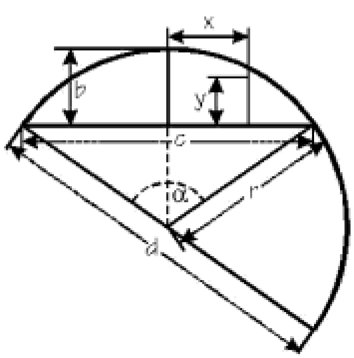
IRREGULAR FIGURE



x = x

$$A = \left(\frac{a}{2} + b + c + d + e + f + g + \frac{h}{2}\right)x$$

PROPERTIES OF THE CIRCLE



Circumference of Circle of Diameter 1 = π = 3.14159265
Circumference of Circle = $2 \pi r$
Diameter of Circle = Circumference X 0.31831
Diameter of Circle of equal periphery as Square = side X 1.27324
Side of Square of equal periphery as Circle = diameter X 0.78540
Diameter of Circle circumscribed about Square = side X 1.41421
Side of Square inscribed in Circle = diameter X 0.70711
Length of Arc = a

Arc, $a = \frac{\pi r \alpha}{180^\circ} = 0.017453\, r \alpha$ **Angle,** $\alpha = \frac{180^\circ a}{\pi r} = 57.29578 \frac{a}{r}$

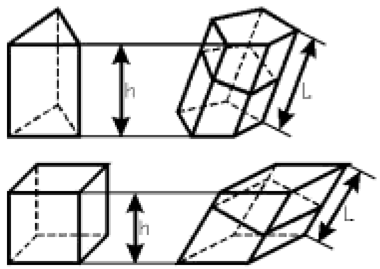
Radius, $r = \frac{4b^2 + c^2}{8b}$ **Diameter,** $d = \frac{4b^2 + c^2}{4b}$ **Chord,** $c = 2\sqrt{2br - b^2} = 2r \sin \frac{\alpha}{r}$

Rise, $b = r - \frac{1}{2}\sqrt{4r^2 - c^2} = \frac{c}{2} \tan \frac{\alpha}{4} = 2r \sin^2 \frac{\alpha}{4}$

Rise, $b = r + y - \sqrt{r^2 - x^2}$ $y = b - r + \sqrt{r^2 - x^2}$ $x = \sqrt{r^2 - (r + y - b)^2}$

Areas of Irregular Figures and Geometric Properties of a Circle

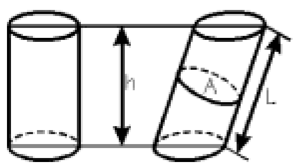
SURFACE AND VOLUME OF SOLIDS



PRISM

(RIGHT OR OBLIQUE, REGULAR OR IRREGULAR, PARALLELOPIPED)

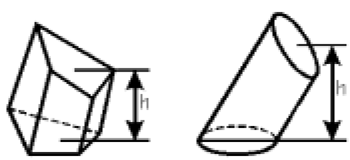
GIVEN	SOUGHT
Perimeter. P, perpendicular to sides; lateral length, L.	Lateral Surface = PL
Area of base, B; perpendicular height, h.	Volume = Bh
Area section perpendicular to sides, A; lateral length, L.	Volume = AL



CYLINDER

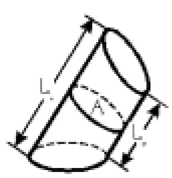
(RIGHT OR OBLIQUE, CIRCULAR OR ELLIPTIC)

GIVEN	SOUGHT
Perimeter of base, P_b; perpendicular height, h.	Lateral Surface = $P_b h$
Perimeter, P, perpendicular to sides; lateral length, L.	Lateral Surface = PL
Area of base, B; perpendicular height, h.	Volume = Bh
Area of section perpendicular to sides, A; lateral length, L.	Volume = AL



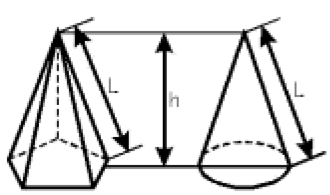
FRUSTRUM OF ANY PRISM OR CYLINDER

GIVEN	SOUGHT
Area of base, B; perpendicular distance from base to center of gravity of opposite face, h.	Volume = Bh



FRUSTRUM OF CYLINDER

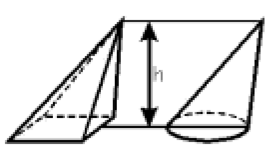
GIVEN	SOUGHT
Area of section perpendicular to sides, A; maximum lateral length, L_1, and minimum, L_2.	Volume = ½ $A(L_1 + L_2)$



PYRAMID OR CONE

(RIGHT AND REGULAR)

GIVEN	SOUGHT
Perimeter of base, P_b; slant height, L.	Lateral Surface = ½ $P_b L$
Area of base, B; perpendicular	Volume = 1/3Bh



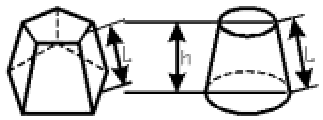
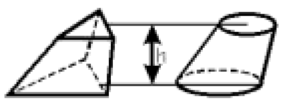
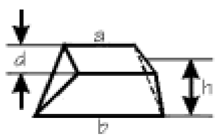
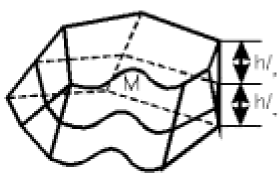
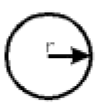
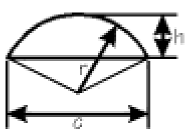
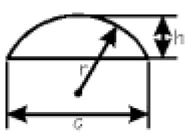
PYRAMID OR CONE

(RIGHT OR OBLIQUE, REGULAR OR IRREGULAR)

GIVEN	SOUGHT
Area of base, B; perpendicular height, h.	Volume = 1/3Bh = 1/3 the volume of prism or cylinder of same base and perpendicular height or ½ the volume of hemisphere of same base and perpendicular height.

Surface and Volume of Solids

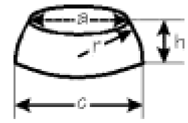
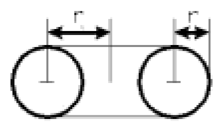
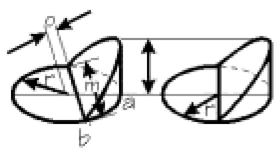
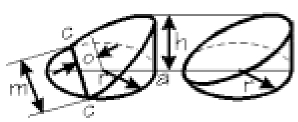
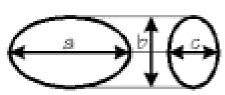
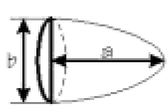
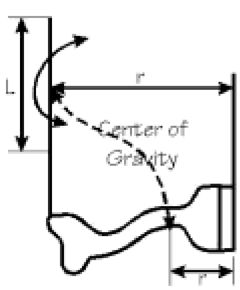
SURFACE AND VOLUME OF SOLIDS

	GIVEN	*SOUGHT*
	**FRUSTRUM FOR PYRAMID OR CONE** (RIGHT AND REGULAR PARALLEL ENDS) Perimeters of base, P_b, and top, P_t; slant height, L. Areas of base, B, and top, T; perpendicular height, h.	Lateral Surface = $\frac{1}{2} L (P_b + P_t)$ Volume = $1/3\, h (B + T + \sqrt{BT})$
	**FRUSTRUM FOR ANY PYRAMID OR CONE** (PARALLEL ENDS) Areas of base, B, and top, T; perpendicular height, h.	Volume = $1/3\, h (B + T + \sqrt{BT})$
	**WEDGE** (PARALLELOGRAM FACE) Length of edges, a and b; perpendicular height, h; perpendicular width, d.	Volume = $\frac{1}{6}\, dh\, (2a + b)$
	**PRISMATOID** Areas of base, B, top, T, and of a section, M, parallel to and midway between base and top; perpendicular height, h.*	Volume = $\frac{1}{6}\, h (B + T + 4M)$
	**SPHERE** Radius, r.	Area = $4\pi r^2$ Volume = $\frac{4}{3}\pi r^3$
	**SPHERICAL SECTOR** Radius, r; length of chord, c; height, h.	Area = $\frac{\pi r}{2}(4h + c)$ Volume = $\frac{2}{3}\pi r^2 h$
	**SPHERICAL SEGMENT** Radius, r; length of chord, c; height, h.	Curved Surface = $2\pi rh = \frac{\pi}{4}(4h^2 + c^2)$ Volume = $\frac{\pi}{3} h^2 (3r - h) = \frac{\pi}{24} h (3c^2 + 4h^2)$

* This formula also applies to any of the foregoing solids with parallel bases, to pyramids, cones, spherical sections and to many solids with irregular surfaces.

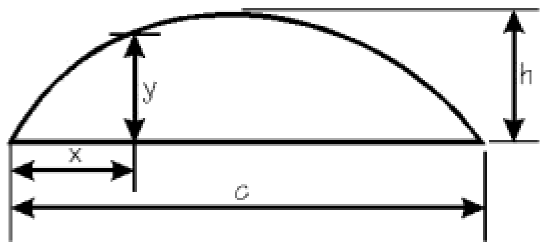
Surface and Volume of Solids

SURFACE AND VOLUME OF SOLIDS

	GIVEN	SOUGHT
	**SPHERICAL ZONE** Radius, r; height, h; diameters, a and c.	Curved Surface = $2\pi rh$ Volume = $\frac{\pi h}{24}(3a^2 + 3c^2 + 4h^2)$
	**RING OF CIRCULAR CROSS SECTION, TORUS** Radius of ring, r, and of cross section, r_1.	Area = $4\pi^2 r r_1$ Volume = $2\pi^2 r r_1^2$
	UNGULA OF RIGHT, REGULAR CYLINDER	
	Base = segment, bab' (smaller than semi-circle). Radius of cylinder, r; height, h; length of chord, m; distance from chord to axis of cylinder, o.	Convex Surface = $(rm - o \cdot \text{arc } bab')\frac{h}{r-o}$ Volume = $\left(\frac{m^3}{12} - o \cdot \text{area } bab'\right)\frac{h}{r-o}$
	Base = semi-circle, Radius of cylinder, r; height, h.	Convex Surface = $2rh$ Volume = $\frac{2}{3} r^2 h$
	Base = segment, cac' (larger than semi-circle). Radius of cylinder, r; height, h; length of chord, m; distance from chord to axis of cylinder, o.	Convex Surface = $(rm + o \cdot \text{arc } cac')\frac{h}{r+o}$ Volume = $\left(\frac{m^3}{12} + o \cdot \text{arc } cac'\right)\frac{h}{r+o}$
	Base = circle, Radius of cylinder, r; height, h.	Convex Surface = $r\pi h$ Volume = $\frac{1}{2} r^2 \pi h$
	**ELLIPSOID** Axes, a, b and c.	Volume = $\frac{1}{6}\pi abc$
	**PARABOLOID** Axes, a, b and c. The ratio of corresponding volumes of cone, paraboloid, sphere and cylinder of equal height is $\frac{1}{3} : \frac{1}{2} : \frac{2}{3} : 1$	Volume = $\frac{1}{8}\pi ab^2$
	**SURFACE AND SOLIDS OF REVOLUTION** When a plane curve or straight line revolves around an axis of revolution in the same plane, a surface of revolution is produced. A solid of revolution results from the rotation of an area around an axis in its plane. Length of curve or straight line, L; normal distance from center of gravity to axis, r; angle of revolution, $\alpha°$; area of revolving plane, A.	Length of arc described by center of gravity = $2\pi r \frac{\alpha}{360}$ Surface of revolution = $2\pi rL \frac{\alpha}{360}$ Volume of solid = $2\pi rA \frac{\alpha}{360}$

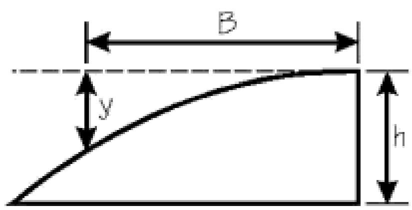
Surface and Volume of Solids

PARABOLA



$$y = \frac{4hx}{c^2}(c - x)$$

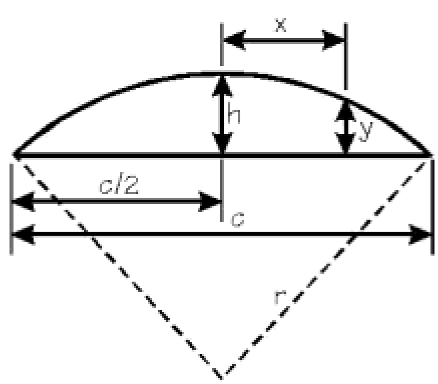
PARABOLIC CROWN ROADWAY



w = width between curbs
h = height of crown
B = distance to any point
y = distance below crown

$$y = \frac{4hB^2}{w^2}$$

CIRCULAR CROWN ROADWAY



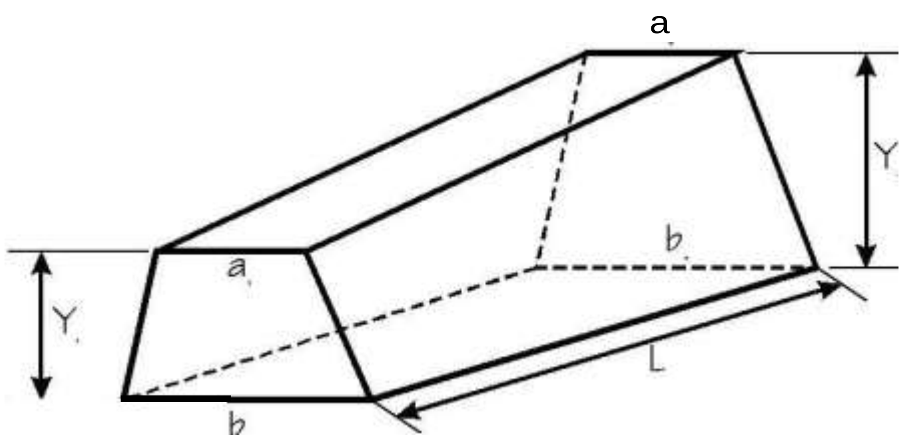
$$r = \frac{4h^2 + c^2}{8h}$$

$$y = h - r + \sqrt{r^2 - x^2}$$

$$y = h + \sqrt{r^2 - x^2} - r$$

Circular and Parabolic Roadway Crown Calculations

RETAINING WALL VOLUME FORMULA



$$\textbf{Volume=}\ \frac{L\ P}{24}$$

L = Distance Between 2 Parallel Faces

$$P=hV+hV+hV$$

$$h_1 = a_1 + b$$

$$V_1 = 2Y_1$$

$$h_2 = a_2 + b$$

$$v_2 = 2\ y_2$$

$$h = h + h_1$$

$$V = V + V$$